

# WAdS<sub>3</sub>/CFT<sub>2</sub> correspondence in presence of bulk massive gravitons

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## Abstract

In a previous paper, it has been shown that the entropy of non-extremal black holes in Warped Anti-de Sitter (WAdS) spaces in massive gravity can be computed microscopically in terms of a dual conformal field theory. Here, we extend this computation to a set of asymptotic boundary conditions that, while still gathering the WAdS<sub>3</sub> black holes, also allow for new solutions that are not locally equivalent to WAdS<sub>3</sub> space, and therefore are associated to the local degrees of freedom of the theory (bulk massive gravitons). After presenting explicit examples of such geometries, we compute the asymptotic charge algebra and show that it is generated by the semi-direct sum of Virasoro algebra and an affine Kac-Moody algebra. The value of the central charge turns out to be exactly the one that leads to reproduce the entropy of the WAdS<sub>3</sub> black holes. This result probes the WAdS<sub>3</sub>/CFT<sub>2</sub> correspondence in presence of bulk gravitons.

# 1 Introduction

Warped Anti-de Sitter spaces (WAdS) are squashed or stretched deformations of 3-dimensional AdS space that preserve the  $SL(2, \mathbb{R}) \times U(1)$  subgroup of the  $AdS_3$  isometries. These spaces happen to be solutions of a large class of theories, including string theory, and appear in several contexts, such as in the Kerr/CFT correspondence [1].

In Ref. [2], a WAdS/CFT correspondence was conjectured, stating that quantum gravity in asymptotically WAdS spaces is dual to a 2-dimensional conformal field theory (CFT). This represents one of the most interesting attempts to extend AdS/CFT correspondence to non-AdS, less symmetric spaces. One of the reasons why WAdS/CFT is interesting is the existence of black hole solutions [3] that are locally equivalent[2] to  $WAdS_3$  and are also asymptotically  $WAdS_3$ . Therefore, the WAdS/CFT correspondence provides a setup to address the holographic description of non-extremal, non-AdS black holes. In a previous paper[4], we showed that, for the particular case of massive gravity, the  $WAdS_3$  black hole entropy could be microscopically described in terms of a dual  $CFT_2$ . More precisely, we showed that the states of the dual theory that corresponds to the black hole configurations organize in highest weight representations of two mutually commuting Virasoro algebras, and that the Cardy formula for the asymptotic growth of such states exactly reproduces the black hole entropy.

A crucial ingredient in the analysis done in Ref. [4] is the asymptotic boundary conditions considered. The question we are concerned with in this paper is whether a similar WAdS/CFT computation can still be carried out for a set of boundary conditions which, on top of the black hole solutions, admit also the presence of bulk gravitons<sup>1</sup>. That is, we will consider a one-parameter family of boundary conditions which, while accommodating the  $WAdS_3$  black holes configurations, also gather non-locally  $WAdS_3$  solutions. The result we will obtain strongly suggests that WAdS/CFT correspondence is still valid in presence of bulk gravitons.

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<sup>1</sup>The term *bulk graviton* is used in contraposition to the term *boundary graviton*, which is usually employed to refer to degrees of freedom associated to the representations of the asymptotic isometries; the latter being also present in theories with no local degrees of freedom.

## 2 WAdS<sub>3</sub> black holes and massive gravity

We are interested in the black hole geometries [3, 5],

$$\begin{aligned} \frac{ds^2}{l^2} = dt^2 + \frac{dr^2}{(\nu^2 + 3)(r - r_+)(r - r_-)} + \left(2\nu r - \sqrt{r_+ r_- (\nu^2 + 3)}\right) dt d\varphi \\ + \frac{r}{4} \left(3(\nu^2 - 1)r + (\nu^2 + 3)(r_+ + r_-) - 4\nu \sqrt{r_+ r_- (\nu^2 + 3)}\right) d\varphi^2, \end{aligned} \quad (1)$$

which at large distance asymptote WAdS<sub>3</sub> space. Metric (1) represents black holes with inner and outer horizons located at  $r_+$  and  $r_-$ , respectively.  $\nu$  is the parameter that controls the stretching deformation, with AdS<sub>3</sub> corresponding to  $\nu = 1$ ;  $l$  is the curvature radius.

Black holes (1) can be constructed by means of global identifications from the WAdS<sub>3</sub> space [2]. This orbifold type construction of these geometries yields two geometrical temperatures, given by the inverse of the identification periods; namely

$$T_L = \frac{\nu^2 + 3}{8\pi l^2} (r_+ + r_- - \frac{1}{\nu} \sqrt{(\nu^2 + 3)r_+ r_-}), \quad T_R = \frac{\nu^2 + 3}{8\pi l^2} (r_+ - r_-). \quad (2)$$

As mentioned before, WAdS<sub>3</sub> spaces and black holes appear in a large class of theories, such as topologically massive gauge theories, string theory, and many others. A minimal set up in which these geometries appear as solutions is 3-dimensional pure gravity, provided one gives to the graviton a mass

$$m^2 = \frac{(20\nu^2 - 3)}{2l^2}, \quad (3)$$

and the cosmological constant takes the value

$$\Lambda = \frac{(4\nu^4 - 48\nu^2 + 9)}{2l^2(20\nu^2 - 3)}. \quad (4)$$

In three dimensions, there is a consistent manner to give mass to the graviton preserving parity invariance. This amounts to consider the higher-order action

$$\mathcal{I} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{m^2} (R_{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2) \right), \quad (5)$$

which is a fully covariant extension of the spin-2 massive Fierz-Pauli theory [6].

The mass ( $M$ ) and angular momentum ( $J$ ) of black holes (1) in massive gravity (5) can be computed by different methods. Their expressions in terms of  $r_-$  and  $r_+$  can be found, for

instance, in Ref. [4]. On the other hand, the entropy of these black holes can be computed with the Wald formula, and in terms of the mass and the angular momentum, it takes the form

$$S_{\text{BH}} = \frac{4\pi l\nu}{(\nu^2 + 3)}(M + \sqrt{M^2 - kJ}), \quad (6)$$

with  $k = 4\nu(3 + \nu^2)/(Gl(20\nu^2 - 3))$ .

### 3 Asymptotic symmetries and charges

Black hole solutions (1) break the  $SL(2, \mathbb{R}) \times U(1)$  isometry group down to  $U(1) \times U(1)$ . The latter group is generated by the two Killing vectors  $\xi^{(1)} = \partial_t$ ,  $\xi^{(2)} = -\partial_\varphi$ . We will see, however, that the asymptotic isometry group is infinite-dimensional. The asymptotic boundary conditions we consider here are defined as follows: We first consider as reference background the metric (1) with  $r_- = r_+ = 0$ ; we denote the metric of such geometry by  $ds_0^2$ . Then, the set of geometries to be considered are defined as those deformations of the form  $ds^2 = ds_0^2 + \delta g_{\mu\nu} dx^\nu dx^\mu$  that respect the following fall-off conditions

$$\begin{aligned} \delta g_{tt} &= \mathcal{O}(r^{-3}), & \delta g_{tr} &= \mathcal{O}(r^{\delta-4}), & \delta g_{t\varphi} &= \mathcal{O}(r^{\delta-1}), \\ \delta g_{rr} &= \mathcal{O}(r^{\delta-4}), & \delta g_{r\varphi} &= \mathcal{O}(r^{\delta-2}), & \delta g_{\varphi\varphi} &= \mathcal{O}(r^\delta). \end{aligned} \quad (7)$$

where  $\delta$  is a real parameter that here we will assumed greater than 1. For  $\delta > 1$ , asymptotic conditions (7) exhibit components that fall off slower than those of [7, 4].

Boundary conditions (7) are similar to those considered in the literature [8] for the parity-odd theory, and it is worthwhile comparing such definition with the asymptotic boundary conditions of Refs. [7, 4]. Fall-off conditions (7) accommodate, in particular, the black hole solutions (1), but also include other solutions: An example of such a solution is given by the Kerr-Schild ansatz

$$ds^2 = ds_0^2 + e^{\omega t} r^{-\frac{2\nu\omega}{3+\nu^2}} h(u) k_\mu k_\nu dx^\mu dx^\nu, \quad (8)$$

with the null vector  $k_\mu$

$$k_\mu dx^\mu = \frac{2}{3 + \nu^2} \frac{dr}{r^2} - d\varphi, \quad (9)$$

$h(u)$  being an arbitrary periodic function of the variable  $u \equiv \varphi + 2/((\nu^2 + 3)r)$  (hereafter, we set  $l = 1$ ) and  $\omega$  being a solution of the polynomial equation

$$P(\nu, \omega) \equiv -4\nu\omega^2 - 6\omega + 10\omega\nu^2 + 16\nu^3 - \omega^3 = 0. \quad (10)$$

Solution (8)-(10) obeys the boundary conditions (7) with  $\delta = -2\nu\omega/(\nu^2 + 3)$  and are not locally equivalent to WAdS<sub>3</sub>.

Asymptotic Killing vectors respecting the set of boundary conditions (7) are given by

$$\begin{aligned}\ell_n &= \frac{4\nu n^2}{(3 + \nu^2)} \frac{1}{r} e^{-in\varphi} \partial_t + \frac{2\nu n^2 - r^2(3 + \nu^2)}{(3 + \nu^2)} \frac{1}{r^2} e^{-in\varphi} \partial_\varphi - inr e^{-in\varphi} \partial_r + \dots \\ t_n &= (e^{in\varphi} + \mathcal{O}(r^{-1})) \partial_t + \dots\end{aligned}\tag{11}$$

where  $n \in \mathbb{Z}$ , and where the ellipses stand for sub-leading terms. In particular, these asymptotic Killing vectors include the exact Killing vectors  $\ell_0 = -\partial_\varphi$  and  $t_0 = \partial_t$ . The full set of asymptotic Killing vectors satisfy the algebra

$$i[\ell_m, \ell_n] = (m - n)\ell_{m+n}, \quad i[\ell_m, t_n] = -nt_{m+n}, \quad i[t_m, t_n] = 0,\tag{12}$$

which is a semi-direct sum of Witt algebra and the loop algebra of  $u(1)$ .

The next step is to compute the charges associated to vectors  $\ell_n$  and  $t_n$ . This computation can be performed as explained in Ref. [4]; that is, by resorting to the Brandt-Barnich-Compère covariant formalism [9, 10], using the metric  $ds_0^2$  as reference background, and the expression of the charges derived in Ref. [11] for massive gravity. If we denote by  $L_n$  and  $P_n$  the charges associated to  $\ell_n$  and  $t_n$  respectively, then the algebra of charges reads

$$\begin{aligned}i\{L_m, L_n\} &= (m - n)L_{m+n} + \frac{c}{12}m^3\delta_{m+n,0}, \\ i\{L_m, P_n\} &= -nP_{m+n}, \quad i\{P_m, P_n\} = \frac{k}{2}m\delta_{m+n,0},\end{aligned}\tag{13}$$

where

$$c = \frac{96\nu^3}{G(20\nu^4 + 57\nu^2 - 9)},\tag{14}$$

and where  $k$  coincides with the value already appearing in (6). This algebra is a semi-direct sum of Virasoro algebra with central charge  $c$  and the affine  $\hat{u}(1)_k$  Kac-Moody algebra of level  $k$ .

In order to organize the states that correspond to the black hole configurations, we define the Sugawara operators [12, 4]

$$L_n^- \equiv \frac{1}{k} \sum_{m \in \mathbb{Z}} : P_{-n-m} P_m : , \quad L_n^+ \equiv L_n^- + L_n.\tag{15}$$

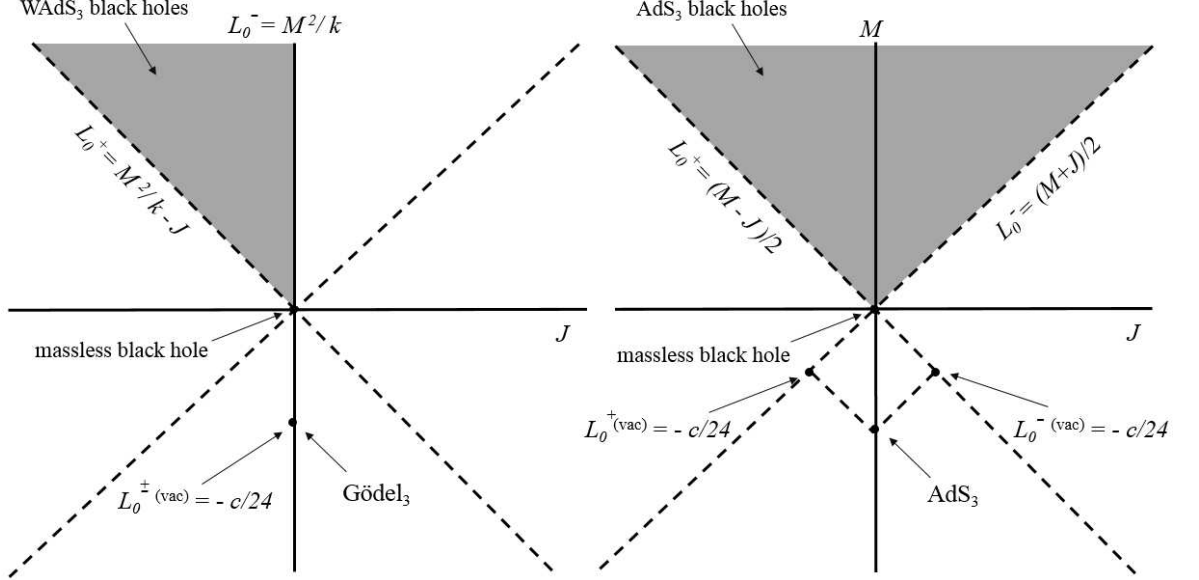


Figure 1: The spectrum of WAdS<sub>3</sub> black holes is qualitatively similar to that of the AdS<sub>3</sub> black holes. The background geometry with respect to which one computes the conserved charges and the asymptotic boundary conditions is the configuration with  $r_+ = r_- = 0$ . On the other hand, the spectrum exhibits a gap with the ground state being the configuration with  $L_0^\pm = -c/24$ ; in the case of AdS<sub>3</sub> black holes this corresponds to AdS<sub>3</sub> space, while for the WAdS<sub>3</sub> black holes this corresponds to Gödel space.

These operators both obey the Virasoro algebra with central charges  $c^- = 1$  and  $c^+ = c - 1$ , respectively; they also satisfy  $[L_n^+, L_m^-] = 0$ . One can also verify that these operators are bounded from below [4] when evaluated on the black hole spectrum (see Fig. 1); namely,

$$L_0^+ = \frac{1}{k}M^2 - J \geq 0, \quad L_0^- = \frac{1}{k}M^2 \geq 0. \quad (16)$$

## 4 Microscopic black hole entropy

The existence of two mutually commuting Virasoro algebras whose highest weight representations are consistent with the black hole spectrum invites to check whether the Cardy entropy formula reproduces the black hole entropy (6). Then, we consider

$$S_{\text{CFT}} = 2\pi\sqrt{\frac{c_{\text{eff}}^-}{6}L_0^-} + 2\pi\sqrt{\frac{c_{\text{eff}}^+}{6}L_0^+}, \quad (17)$$

where  $c_{\text{eff}}^\pm \equiv -24L_0^{\pm(\text{vac})}$  are the effective central charges,  $L_0^{\pm(\text{vac})}$  being the minimum values that charges  $L_0^\pm$  take in the spectrum. That is, the values  $L_0^{\pm(\text{vac})}$  are the charges evaluated on the geometry that is to be identified as the ground state of the black hole spectrum (i.e. the vacuum). Such geometry has to exhibit the full  $SL(2, \mathbb{R}) \times U(1)$  isometry group, and can be argued to be the timelike WAdS<sub>3</sub> space, which corresponds to the 3-dimensional section of Gödel spacetime [13, 4]. The latter is precisely the geometry that reduces to global AdS<sub>3</sub> when  $\nu = 1$ , and its geometry corresponds to [14]

$$L_0^{\pm(\text{vac})} = -\frac{c}{24}. \quad (18)$$

This implies that the effective central charges associated to the Virasoro algebras generated by operators  $L_n^\pm$  are given by  $c_{\text{eff}}^\pm = c$ . This explains the fact that black hole entropy (6) can be expressed in terms of the geometric temperatures (2) by

$$S_{\text{BH}} = \frac{\pi^2}{3}(c_{\text{eff}}^- T_L + c_{\text{eff}}^+ T_R) = \frac{\pi^2}{3}c(T_L + T_R), \quad (19)$$

with the value of  $c$  conjectured in Ref. [15].

With (18), one finds that Cardy formula (17) exactly matches the black hole entropy (6); namely one finds  $S_{\text{CFT}} = S_{\text{BH}}$ . This result generalizes our previous result of [4] to the one-parameter family of boundary conditions (7), and can be regarded as a consistency check of WAdS/CFT correspondence in the case of boundary conditions that accommodate bulk gravitons.

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## References

- [1] M. Guica, T. Hartman, W. Song, and A. Strominger, “The Kerr/CFT Correspondence,” *Phys.Rev.* **D80** (2009) 124008, [arXiv:hep-th/0809.4266](#).
- [2] D. Anninos, W. Li, M. Padi, W. Song, and A. Strominger, “Warped AdS(3) Black Holes,” *JHEP* **03** (2009) 130, [arXiv:hep-th/0807.3040](#).
- [3] K. Ait Moussa, G. Clément, C. Leygnac, “The Black holes of topologically massive gravity,” *Class. Quant. Grav.* **20** (2003) L277-L283, [arXiv:gr-qc/0303042](#).
- [4] L. Donnay and G. Giribet, “Holographic entropy of Warped-AdS<sub>3</sub> black holes,” *JHEP* **06** (2015) 099, [arXiv:hep-th/1504.05640](#).
- [5] G. Clément, “Warped AdS(3) black holes in new massive gravity,” *Class. Quant. Grav.* **26** (2009) 105015, [arXiv:hep-th/0902.4634](#).
- [6] E. A. Bergshoeff, O. Hohm and P. K. Townsend, “Massive Gravity in Three Dimensions,” *Phys. Rev. Lett.* **102** (2009) 201301, [arXiv:hep-th/0901.1766](#).
- [7] G. Compère and S. Detournay, “Boundary conditions for spacelike and timelike warped AdS<sub>3</sub> spaces in topologically massive gravity,” *JHEP* **0908** (2009) 092, [arXiv:hep-th/0906.1243](#).
- [8] M. Henneaux, C. Martínez, R. Troncoso, “Asymptotically warped anti-de Sitter spacetimes in topologically massive gravity,” *Phys. Rev.* **D84** (2011) 124016, [arXiv:hep-th/1108.2841](#).
- [9] G. Barnich and F. Brandt, “Covariant theory of asymptotic symmetries, conservation laws and central charges,” *Nucl.Phys.* **B633** (2002) 3–82, [arXiv:hep-th/0111246](#).
- [10] G. Barnich and G. Compère, “Surface charge algebra in gauge theories and thermodynamic integrability,” *J.Math.Phys.* **49** (2008) 042901, [arXiv:gr-qc/0708.2378](#).
- [11] S. Nam, J. D. Park and S. H. Yi, “Mass and Angular momentum of Black Holes in New Massive Gravity,” *Phys. Rev.* **D 82** (2010) 124049, [arXiv:1009.1962](#).



- [12] M. Blagojević and B. Cvetković, “Asymptotic structure of topologically massive gravity in spacelike stretched AdS sector,” *JHEP* **0909** (2009) 006, [arXiv:0907.0950 \[gr-qc\]](#).
- [13] S. Detournay, T. Hartman, and D. M. Hofman, “Warped Conformal Field Theory,” *Phys. Rev.* **D86** (2012) 124018, [arXiv:hep-th/1210.0539](#).
- [14] L. Donnay, J.J. Fernandez-Melgarejo, G. Giribet, A. Goya and E. Lavia, “Conserved charges in timelike warped  $\text{AdS}_3$  spaces,” *Phys. Rev.* **D91** (2015) 125006, [arXiv:hep-th/1504.05212](#).
- [15] Gaston Giribet, Andrés Goya, “The Brown-York mass of black holes in Warped Anti-de Sitter space,” *JHEP* **03** (2013) 130, [arXiv:hep-th/1212.2100](#).